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Effects of the scalar FCNC in $b \to sl^+l^-$ transitions and supersymmetry

P.H. Chankowski, Ł. Sławianowska

Institute of Theoretical Physics, Warsaw University, Hoża 69, 00-681 Warsaw, Poland

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Abstract. We investigate the potential effects of the scalar flavor changing neutral currents that are generated e.g. in supersymmetry with $\tan\beta \gg 1$ in the $b \to sl^+l^-$ transitions. Using the experimental upper limit on $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ we place stringent model independent constraints on the impact these currents may have on the rates $\mathrm{BR}(B \to X_s\mu^+\mu^-)$ and $\mathrm{BR}(B \to K\mu^+\mu^-)$. We find that in the first case, contrary to the claim made recently in the literature, the maximal potential effects are always smaller than the uncertainty of the standard model NNLO prediction, that is of order 5–15%. In the second case, the effects can be large, but the experimental errors combined with the unsettled problems associated with the relevant form factors do not allow for any firm conclusion about the detectability of a new physics signal in this process. In supersymmetry the effects of the scalar flavor changing neutral currents are further constrained by the experimental lower limit on the B_s^0–$\bar{B}_s^0$ mass difference, so that most likely no detectable signal of the supersymmetry generated scalar flavor changing neutral currents in the processes $B \to X_s\mu^+\mu^-$ and $B \to K\mu^+\mu^-$ is possible.

1 Introduction

Rare processes involving the b-quark, intensively studied at present in several experiments (BaBar, BELLE, Tevatron), play an important role in supersymmetry (SUSY) searches via virtual effects of the new particles. This is because in the minimal supersymmetric extension (MSSM) of the standard model (SM) the Yukawa couplings of the b-quark to some of the superpartners of the known particles and/or to the Higgs bosons can be strong enough to produce measurable effects. A celebrated example is the radiative decay $\bar{B} \to X_s\gamma$ whose experimentally measured rate [1] agrees very well with the SM prediction [2] and, consequently, puts constraints on the MSSM parameter space. These constraints become particularly stringent if the ratio $v_u/v_d \equiv \tan\beta$ of the vacuum expectation values of the two Higgs doublets is large, that is when the coupling of the right chiral b-quark to charginos and the top squarks is enhanced: agreement with the experimental value can then be obtained either if all these sparticles as well as the charged Higgs boson H^+ are sufficiently heavy (in which case there is little hope to detect their virtual effects also in other rare processes), or if the virtual chargino–stop contribution to the $b \to s\gamma$ amplitude cancels against the top–charged Higgs boson contribution. The latter solution requires of course a certain amount of fine tuning, which becomes, however, of tolerable magnitude for $M_{H^+} \gtrsim 200\,\mathrm{GeV}$ and sparticles weighing not less than a few hundreds GeV.

A very interesting feature of the large $\tan\beta$ SUSY scenario is the generation at one loop of the $(\tan^2\beta)$-enhanced flavor violating (FV) couplings of the neutral Higgs bosons, A^0 (the CP-odd one) and H^0 (the heavier CP-even one), to the down-type quarks [3]. Being operators of dimension four, these couplings remain unsuppressed even for heavy superpartners of the known particles (gluinos, squarks, charginos). If the flavor violation is minimal (the so-called MFV SUSY), that is if the Cabbibo–Kobayashi–Maskawa (CKM) matrix is the only source of flavor and CP violation, the FV couplings of A^0 and H^0 are very sensitive to the mixing of the left and right top squarks. (Induced by these couplings FV decays of the neutral MSSM Higgs bosons have been investigated in [4].) The exchanges of the neutral Higgs bosons generate then $|\Delta F| = 1$ [5–7] and $|\Delta F| = 2$ [8,9] dimension six operators which contribute to the $b \to sl^+l^-$ and $b\bar{s} \to \bar{b}s$ [9] transitions. For A^0 and H^0 not much heavier than the electroweak scale these operators, called because of their Lorentz structure the scalar operators, can significantly change the predictions of the SM.

Phenomenological consequences of the scalar operators have been analyzed in several papers [5–7,10–22] both in supersymmetry with minimal (MFV) and non-minimal flavor violation. In particular, it has been shown [7,12–14,16] that even in the MFV SUSY the effects of the scalar operators originating from the FV couplings of H^0 and A^0 can, for large mixing of the left and right chiral top squarks, increase $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ and $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$

by 3–4 orders of magnitude. The upper bound on the first of these branching fractions set recently by CDF [23],

$$\mathrm{BR}(B_s^0 \to \mu^+\mu^-) < 0.95 \times 10^{-6} \qquad \text{at } 90\% \text{ C.L.} \quad (1)$$

(which improves the previous limit $\mathrm{BR}(B_s^0 \to \mu^+\mu^-) < 2 \times 10^{-6}$ [24]), puts therefore on the MSSM parameter space a non-trivial constraint which is to a large extent complementary to the one imposed by the measurement of $\mathrm{BR}(\bar{B} \to X_s\gamma)$. On the other hand, as shown in [25], a measurement of the $B_s^0 \to \mu^+\mu^-$ signal at the Tevatron Run II, possible if $\mathrm{BR}(B_s^0 \to \mu^+\mu^-) \gtrsim 2 \times 10^{-8}$, would rule out such models of the soft SUSY breaking terms generation like anomaly and gaugino mediation as well as gauge mediation scenarios with a low messenger scale and a small number of messenger fields.

The impact of the FV couplings of H^0 and A^0 on the $|\Delta F| = 2$ transitions $B_s^0 \leftrightarrow \bar{B}_s^0$, $B_d^0 \leftrightarrow \bar{B}_d^0$ was analyzed within the MFV SUSY in [9,17,18,20]. It was found that the contribution of the $|\Delta F| = 2$ scalar operators constructed out of these couplings to the amplitude of the B_s^0–$\bar{B}_s^0$ mixing is negative and can be very large (B_d^0–$\bar{B}_d^0$ mixing is affected negligibly). Part of the parameter space corresponding to $\tan\beta \gg 1$, light H^0 and A^0 and substantial stop mixing, allowed by the experimental limit on $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ then available, was eliminated by the condition that the calculated B_s^0–$\bar{B}_s^0$ mass difference ΔM_s is not smaller than the experimental lower bound $\Delta M_s \gtrsim 14/\mathrm{ps}$ [26].

Even with the new bound (1) the constraints on the MSSM parameter space imposed by the B_s^0–$\bar{B}_s^0$ mixing are in some cases stronger than the ones stemming from the dimuon channel.

The effects of the scalar operators in the exclusive transitions $\bar{B} \to K\mu^+\mu^-$ and $\bar{B} \to K^*\mu^+\mu^-$ have been investigated in [10,13]. Their impact on $\mathrm{BR}(\bar{B} \to K^*\mu^+\mu^-)$ has been found to be very small. On the other hand, potential effects of the scalar operators in $\bar{B} \to K\mu^+\mu^-$ could be quite sizable in principle, but the experimental limit $\mathrm{BR}(B^+ \to K^+\mu^+\mu^-) < 5.2 \times 10^{-6}$ [27] available at that time was too weak to provide constraints stronger than the experimental upper limit for $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$. Finally, the effects of the scalar operators in the inclusive decay rate $\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-)$ have been taken into account in several papers devoted to the general investigation of the potential SUSY effects in radiative B decays or in the studies of the specific SUSY scenarios like the minimal SUGRA but have not been directly confronted with the bounds provided by the $B_s^0 \to \mu^+\mu^-$ decay and B_s^0–$\bar{B}_s^0$ mixing.

In this paper we fill this gap. We begin in Sect. 2 by recalling the NNLO predictions of the SM for $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ and $\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-)$ improving slightly in the latter case the estimates of the theoretical uncertainties compared to those given in [28]. Then in Sect. 3, following [13], we assess in a model independent way how big effects of the scalar operators in the $\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-)$ and in the $\mathrm{BR}(\bar{B} \to K\mu^+\mu^-)$ decays are still allowed by the CDF bound $\mathrm{BR}(B_s^0 \to \mu^+\mu^-) < 0.95 \times 10^{-6}$. We show, in particular, that the huge effects of the scalar operators found recently in $\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-)$ in [29] are excluded by these constraints. The results of Sect. 3 are valid generally, independently of the mechanism that generates the scalar operators. Finally, in Sect. 4 we concentrate on scalar operators in the MFV version of the MSSM (in which the squark mass matrices are aligned with the quark ones; see [20] for more detailed explanations) and specify the maximal effects of the scalar operators in $\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-)$ and in $\mathrm{BR}(\bar{B} \to K\mu^+\mu^-)$ allowed by the experimental limits on both, the $B_s^0 \to \mu^+\mu^-$ rate and the B_s^0–$\bar{B}_s^0$ mass difference. We summarize the situation in the last section.

2 $b \to sl^+l^-$ and $b \to dl^+l^-$ transitions in the SM

Under the assumption of minimal flavor violation, the effective Hamiltonian describing the $b \to sl^+l^-$ ($b \to dl^+l^-$) and $b \to s\gamma$ transitions takes the form [30]

$$\mathcal{H}^{\mathrm{eff}} = -2\sqrt{2}G_{\mathrm{F}}V_{ts}^{\mathrm{eff}*}V_{tb}^{\mathrm{eff}} \times \left(\sum_{X=1}^{10} C_X(\mu)\mathcal{O}_X(\mu) + \sum_{l=e,\mu,\tau}\sum_{X=S,P} C_X^l(\mu)\mathcal{O}_X^l(\mu)\right), \quad (2)$$

with the following set of operators $\mathcal{O}_X^{(l)}$ [30,31,15]:

$$\begin{aligned}
\mathcal{O}_{1c} &= (\bar{s}_{\mathrm{L}}\gamma^\mu T^a c_{\mathrm{L}})(\bar{c}_{\mathrm{L}}\gamma_\mu T^a b_{\mathrm{L}}),\\
\mathcal{O}_{2c} &= (\bar{s}_{\mathrm{L}}\gamma^\mu c_{\mathrm{L}})(\bar{c}_{\mathrm{L}}\gamma_\mu b_{\mathrm{L}}),\\
\mathcal{O}_3 &= (\bar{s}_{\mathrm{L}}\gamma^\mu b_{\mathrm{L}})\sum_{q=u,d,s,c,b}(\bar{q}\gamma_\mu q),\\
\mathcal{O}_4 &= (\bar{s}_{\mathrm{L}}\gamma^\mu T^a b_{\mathrm{L}})\sum_{q=u,d,s,c,b}(\bar{q}\gamma_\mu T^a q),\\
\mathcal{O}_5 &= (\bar{s}_{\mathrm{L}}\gamma^\mu\gamma^\nu\gamma^\lambda b_{\mathrm{L}})\sum_{q=u,d,s,c,b}(\bar{q}\gamma_\mu\gamma_\nu\gamma_\lambda q),\\
\mathcal{O}_6 &= (\bar{s}_{\mathrm{L}}\gamma^\mu\gamma^\nu\gamma^\lambda T^a b_{\mathrm{L}})\sum_{q=u,d,s,c,b}(\bar{q}\gamma_\mu\gamma_\nu\gamma_\lambda T^a q),\\
\mathcal{O}_7 &= \frac{e}{g_{\mathrm{s}}^2}(\bar{s}_{\mathrm{L}}\sigma^{\mu\nu}b_{\mathrm{R}})F_{\mu\nu}, \qquad (3)\\
\mathcal{O}_8 &= \frac{1}{g_{\mathrm{s}}}(\bar{s}_{\mathrm{L}}\sigma^{\mu\nu}T^a b_{\mathrm{R}})G_{\mu\nu}^a,\\
\mathcal{O}_9 &= \frac{e^2}{g_{\mathrm{s}}^2}(\bar{s}_{\mathrm{L}}\gamma^\mu b_{\mathrm{L}})\sum_l(\bar{l}\gamma^\mu l),\\
\mathcal{O}_{10} &= \frac{e^2}{g_{\mathrm{s}}^2}(\bar{s}_{\mathrm{L}}\gamma^\mu b_{\mathrm{L}})\sum_l(\bar{l}\gamma^\mu\gamma^5 l),\\
\mathcal{O}_S^l &= \frac{e^2}{g_{\mathrm{s}}^2}(\bar{s}_{\mathrm{L}}b_{\mathrm{R}})(\bar{l}l),\\
\mathcal{O}_P^l &= \frac{e^2}{g_{\mathrm{s}}^2}(\bar{s}_{\mathrm{L}}b_{\mathrm{R}})(\bar{l}\gamma^5 l)
\end{aligned}$$

and $\mathcal{O}_{1u}$, $\mathcal{O}_{2u}$ obtained from $\mathcal{O}_{1c}$ and $\mathcal{O}_{2c}$ by the replacement $c \to u$, and the Wilson coefficients $C_X(\mu)$ organized as [30]

$$C_X(\mu) = C_X^{(0)}(\mu) + \frac{g_s^2(\mu)}{(4\pi)^2}C_X^{(1)}(\mu) + \frac{g_s^4(\mu)}{(4\pi)^4}C_X^{(2)}(\mu) + \ldots \quad (4)$$

The coefficients C_X computed at some scale $\mu_0 \sim m_t$ are subsequently evolved down to the scale $\mu_b \sim m_b$, where their matrix elements between the hadronic initial and final states of the process under investigation are computed either by lattice methods or perturbatively to the required accuracy in $\alpha_s(\mu_b) = g_s^2(\mu_b)/4\pi$. At the matching scale μ_0 only the coefficients of the operator $\mathcal{O}_2$ starts at order $(\alpha_s)^0$; for the remaining ones $C_X^{(0)}(\mu_0) = 0$.

2.1 $B^0_{s,d} \to \mu^+\mu^-$ in the SM

In the SM the Wilson coefficients C_S and C_P are negligible and the only operator relevant for the $B^0_{s,d} \to l^+l^-$ transitions is $\mathcal{O}_{10}$. Its Wilson coefficients $C_{10}^{(1)}$ and $C_{10}^{(2)}$ at the matching scale are known [32,31]. Since the quark part of $\mathcal{O}_{10}$ is a (partially) conserved chiral current, the QCD evolution of C_{10} is simple, i.e. $C_{10}(\mu_b) = [\alpha_s(\mu_b)/\alpha_s(\mu_0)]C_{10}(\mu_0)$. This leads to the well-known prediction [33]

$$\mathrm{BR}(B^0_q \to l^+l^-) = \frac{\tau(B^0_q)}{\pi} M_{B^0_q} \times \left(\frac{G_\mathrm{F}\alpha_\mathrm{em}\hat{F}_{B_q} m_l}{4\pi \sin^2\theta_\mathrm{W}}\right)^2 \sqrt{1 - 4\frac{m_l^2}{M^2_{B^0_q}}} |V^*_{tq}V_{tb}|^2 |Y(x_t)|^2 \;, \tag{5}$$

where

$$\frac{1}{\sin^2\theta_\mathrm{W}} Y(x_t) = C_{10}^{(1)}(x_t) + \frac{g_s^2(\mu_0)}{16\pi^2} C_{10}^{(2)}(x_t,\mu_0), \tag{6}$$

and $x_t = (m_t^{\overline{\mathrm{MS}}}(\mu_0)/M_W)^2$. $C_{10}^{(1)}(x_t)$ is given by the function $Y_0(x_t)$, which can be found e.g. in [33] and $C_{10}^{(2)}(x_t,\mu_0)$ has been computed in [32] (it can also be extracted from [31]). For $m_t^{\overline{\mathrm{MS}}}(m_t) = (166 \pm 5)\,\mathrm{GeV}$, $\alpha_s(M_Z) = 0.119$ and using $\mu_0 = m_t = 174.3\,\mathrm{GeV}$

$$Y(x_t) = \eta\ (0.971 \pm 0.046), \tag{7}$$

where $\eta = 1.01$ accounts for the effects of $C_{10}^{(2)}$. For $\sin^2\theta_\mathrm{W} = 0.23124$ and $\alpha_\mathrm{em} = 1/128$ this gives

$$\begin{aligned}
&\mathrm{BR}(B^0_s \to \mu^+\mu^-) = (3.64 \pm 0.33) \times 10^{-9} \\
&\quad\times \left(\frac{\tau_{B^0_s}}{1.461\,\mathrm{ps}}\right)\left(\frac{\hat{F}_{B_s}}{238\,\mathrm{MeV}}\right)^2\left(\frac{|V_{ts}|}{0.04}\right)^2, \\
&\mathrm{BR}(B^0_d \to \mu^+\mu^-) = (1.39 \pm 0.13) \times 10^{-10} \\
&\quad\times \left(\frac{\tau_{B^0_d}}{1.542\,\mathrm{ps}}\right)\left(\frac{\hat{F}_{B_d}}{203\,\mathrm{MeV}}\right)^2\left(\frac{|V_{td}|}{0.009}\right)^2,
\end{aligned} \tag{8}$$

where the errors correspond to the variation of $m_t^{\overline{\mathrm{MS}}}(m_t)$. The dominant uncertainties of the SM predictions (of order $\sim^{+28}_{-24}\%$ and $\sim^{+40}_{-30}\%$ in the case of the B^0_s and B^0_d decays, respectively) come from the factors $\hat{F}_{B_s} = (238 \pm 31)\,\mathrm{MeV}$ and $\hat{F}_{B_d} = (203 \pm 27^{+0}_{-20})\,\mathrm{MeV}$ [34] that parameterize the non-perturbative hadronic matrix element of the $\mathcal{O}_{10}$ operator. The uncertainty associated with $\Delta m_t^{\overline{\mathrm{MS}}}(m_t) = 5\,\mathrm{GeV}$, with the electromagnetic corrections and, in the case the B^0_d decay with the value of $|V_{td}|$, are much smaller.

The corresponding branching ratios for the e^+e^- channel are suppressed by the factor $(m_e/m_\mu)^2 \sim 2\times10^{-5}$ and, hence, are unmeasurably small; those for the $\tau^+\tau^-$ channel are enhanced by $(m_\tau/m_\mu)^2 \sim 283$ but tauons are very difficult to identify experimentally.

The present experimental bounds $\mathrm{BR}(B^0_s \to \mu^+\mu^-) < 0.95 \times 10^{-6}$ [23] and $\mathrm{BR}(B^0_d \to \mu^+\mu^-) < 1.6 \times 10^{-7}$ [35, 23] are 3 orders of magnitude above the predictions (8) and still leave a lot of room for new physics.

2.2 The inclusive process $\bar{B} \to X_s l^+l^-$ in the SM

The general formula for the differential width of the $B \to X_s l^+l^-$ decay reads [15,29]

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}s}\Gamma(\bar{B} \to X_s\mu^+\mu^-) \\
&= \frac{G_\mathrm{F}^2\alpha_\mathrm{em}^2 m_b^5}{768\pi^5}|V^*_{tq}V_{tb}|^2\lambda^{1/2}(1,r_s,s)\lambda^{1/2}(1,r_s/s,r_s/s) \\
&\times \Big\{ G_c(s) + f_1(s)G_1(s)\left|\tilde{C}_9^\mathrm{eff}(s,\mu_b)\right|^2 \\
&\quad + f_2(s)G_1(s)\left|\tilde{C}_{10}^\mathrm{eff}(s,\mu_b)\right|^2 \\
&\quad + f_3(s)G_2(s)\left|\tilde{C}_7^\mathrm{eff}(s,\mu_b)\right|^2 \\
&\quad + f_4(s)G_3(s)\mathrm{Re}\left(\tilde{C}_7^\mathrm{eff}(s,\mu_b)\tilde{C}_9^{\mathrm{eff}*}(s,\mu_b)\right) \\
&\quad + f_5(s)\left|C_S^{(1)}(\mu_b)\right|^2 + f_6(s)\left|C_P^{(1)}(\mu_b)\right|^2 \\
&\quad + f_7(s)\mathrm{Re}\left(\tilde{C}_{10}^\mathrm{eff}(s,\mu_b)C_P^{(1)*}(\mu_b)\right)\Big\},
\end{aligned} \tag{9}$$

where $s = q^2/m_b^2$ is the "reduced" invariant mass of the lepton pair and

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc\;. \tag{10}$$

The function $G_c(s,\lambda_1,\lambda_2)$ accounting for the $1/m_c^2$ non-perturbative contribution has been found in [36]. The $1/m_b^2$ non-perturbative contributions summarized by the functions $G_i(s,\lambda_1,\lambda_2)$ have been calculated using the heavy quark expansion technique in [37,38]. The functions $G_c(s,\lambda_1,\lambda_2)$ and $G_i(s,\lambda_1,\lambda_2)$, which depend on the parameters $\lambda_1 \approx -0.2\,\mathrm{GeV}^2$, $\lambda_2 = 0.12\,\mathrm{GeV}^2$ are given in (29)–(31) of [28]. Finally,[1]

$$f_1(s) = s(1 + r_s - s)\lambda(1, r_s/s, r_s/s)$$

[1] The functions $f_3(s)$ and $f_7(s)$ differ from the corresponding expressions in [29]. Due to the extra piece $-s^2$ the function $f_3(s)$ as given here reproduces in the limit $m_s = 0$ the result obtained in earlier papers for the coefficient of $|\tilde{C}_7^\mathrm{eff}|^2$. We also confirm that the sign of $f_7(s)$ is as in the earlier papers [15] (opposite to the one in [29]).

$$
\begin{aligned}
&\quad + (1 - r_s + s)(1 - r_s - s)(1 + 2r_s/s) \\
&\quad + 6r_l(1 + r_s - s), \\
f_2(s) &= s(1 + r_s - s)\lambda(1, r_s/s, r_s/s) \\
&\quad + (1 - r_s + s)(1 - r_s - s)(1 + 2r_s/s) \\
&\quad - 6r_l(1 + r_s - s), \\
f_3(s) &= (4/s)(1 + 2r_s/2) \\
&\quad \times \left[2(1 + r_s)(1 - r_s)^2 - s(1 + 14r_s + r_s^2) - s^2\right], \\
f_4(s) &= 12(1 + 2r_s/s)\left[(1 - r_s)^2 - s(1 + r_s)\right], \\
f_5(s) &= \frac{3}{2}(1 + r_s - s)(s - 4r_l), \\
f_6(s) &= \frac{3}{2}(1 + r_s - s)s, \\
f_7(s) &= 6\sqrt{r_l}(1 - r_s - s),
\end{aligned}
\tag{11}
$$

where $r_l = m_l^2/m_b^2$, $r_s = m_s^2/m_b^2$. In the NNLO approximation the coefficients $\tilde{C}_7^{\rm eff}(s, \mu_b)$, $\tilde{C}_9^{\rm eff}(s, \mu_b)$ and $\tilde{C}_{10}^{\rm eff}(s, \mu_b)$ summarizing the effects of the QCD running from the scale $\mu_0 \sim m_t$ down to the scale $\mu_b \sim m_b$ and the matrix elements of the relevant operators from the list (3) can be compactly written as [39]:

$$
\begin{aligned}
\tilde{C}_7^{\rm eff}(s, \mu_b) &= \left(1 + \frac{\alpha_{\rm s}(\mu_b)}{\pi}\omega_7(s)\right) A_7 \\
&- \frac{\alpha_{\rm s}(\mu_b)}{4\pi}\left(C_1^{(0)}F_1^{(7)}(s) + C_2^{(0)}F_2^{(7)}(s) + C_8^{(1)}F_8^{(7)}(s)\right), \\
\tilde{C}_9^{\rm eff}(s, \mu_b) &= \left(1 + \frac{\alpha_{\rm s}(\mu_b)}{\pi}\omega_9(s)\right) \\
&\quad \times \left[A_9 + T_9 g(m_c^2/m_b^2, s) + U_9 g(1, s) + W_9 g(0, s)\right] \\
&- \frac{\alpha_{\rm s}(\mu_b)}{4\pi}\left(C_1^{(0)}F_1^{(9)}(s) + C_2^{(0)}F_2^{(9)}(s) + C_8^{(1)}F_8^{(9)}(s)\right), \\
\tilde{C}_{10}^{\rm eff}(s, \mu_b) &= \left(1 + \frac{\alpha_{\rm s}(\mu_b)}{\pi}\omega_9(s)\right) A_{10},
\end{aligned}
\tag{12}
$$

where A_i, T_9, U_9, W_9 and the function $g(z, s)$ can be found in [31] and the explicit formulae for the functions $F_j^{(i)}(s)$ and $\omega_i(s)$, valid for $s \lesssim 0.25$, are given in [39].[2] The Wilson coefficients $C_1^{(0)}$, $C_2^{(0)}$ and $C_8^{(1)}$ can be found e.g. in (E.9) of [41]. One should also remember to expand the formula (9) only up to terms of order $\alpha_{\rm s}(\mu_b)$ and to replace $\omega_7(s)$ and $\omega_9(s)$ by $\omega_{79}(s)$ in the interference term. Inclusion to $\tilde{C}_7^{\rm eff}$, $\tilde{C}_9^{\rm eff}$ and $\tilde{C}_{10}^{\rm eff}$ of the $\mathcal{O}(\alpha_{\rm s}(\mu_b))$ corrections to the matrix elements[3] of the relevant operators significantly decreases the dependence of the final result on the renormalization scale μ_b [39]. Since similar $\mathcal{O}(\alpha_{\rm s}(\mu_b))$ corrections to the matrix elements of the operators $\mathcal{O}^l_{S,P}$ are not known at present, their contribution to the rates of the inclusive $B \to X_s l^+ l^-$ processes has the uncertainty associated with the choice of the scale μ_b larger than those of the contributions of the remaining operators.

In order to get rid of the factor m_b^5 in the formula (9), not introducing at the same time the large uncertainty associated with the value of the charm quark mass, we follow the trick proposed in [41] and normalize the rate to the width of the charmless semileptonic decay

$$
\frac{{\rm dBR}(\bar{B} \to X_s\mu^+\mu^-)}{{\rm d}s} = \frac{{\rm BR}(\bar{B} \to X_c e\nu_e)}{C} \times \frac{\frac{\rm d}{{\rm d}s}\Gamma(\bar{B} \to X_s\mu^+\mu^-)}{\frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2\left(1 - \frac{2\alpha_{\rm s}(m_b)}{3\pi}h(0)\right)\left(1 + \frac{\lambda_1}{2m_b^2} - \frac{9\lambda_2}{2m_b^2}\right)},
\tag{13}
$$

where the function $h(z)$ is given e.g. by the formula (48) of [31] and the factor C

$$
C \equiv \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma(\bar{B} \to X_c e\nu_e)}{\Gamma(\bar{B} \to X_u e\nu_e)}
\tag{14}
$$

has been calculated in [41]: $C = 0.575 \times (1 \pm 0.01_{\rm pert} \pm 0.02_{\lambda_1} \pm 0.02_{\Delta}) = 0.575 \times (1 \pm 0.03)$. To remain conservative we will double this uncertainty and use $C = 0.575 \times (1 \pm 0.06)$. The poorly known non-perturbative parameter λ_1 approximately cancels out between the numerator and the denominator. With this trick the residual dependence on $z = m_c^2/m_b^2$ is negligible for m_c^2/m_b^2 varying between 0.27 and 0.31; the uncertainty of the differential branching fraction arising from the normalization is then dominated by the $\sim \pm 6\%$ uncertainty of the factor C (14). It is therefore much smaller than the uncertainty of order $\pm 15\%$ attributed to the differential branching fraction normalized directly to ${\rm BR}(\bar{B} \to X_c e\nu_e)$ in [28] by varying m_c^2/m_b^2 in the range 0.25–0.33.

The dominant source of uncertainty remains the dependence on μ_b which for $s < 0.25$ is estimated (by changing μ_b between 2.5 GeV and 10 GeV) to be of order $\pm 7\%$ [39]. Of comparable magnitude can be however also the uncertainty related to the electromagnetic corrections to the running (and their mixing with others) of the $\mathcal{O}_9$ and $\mathcal{O}_{10}$ operators, which is unknown at present.[4] A simple estimate of this effect is obtained by varying $\alpha_{\rm em}$ in the formula (9) between 1/128 and 1/133. This suggests an additional $\sim 8\%$ uncertainty of the predicted branching ratio. Finally, the parametric uncertainty related to the variation of $m_t^{\overline{\rm MS}}(m_t) = (166 \pm 5)$ GeV is of order $\pm(6$–$7)\%$.

The differential rate (9) can be integrated over various domains of s. The most reliable theoretical predictions are obtained for $0.05 < s < 0.25$ because for this range the non-perturbative effects associated with the $\bar{c}c$ resonances are small and the NNLO calculation is complete. For this region, using $m_t^{\overline{\rm MS}}(m_t) = 166$ GeV, $m_b = 4.8$ GeV, $\alpha_{\rm em} = 1/128$ and $|V_{ts}V_{tb}^*/V_{cb}| = 0.976$ we get

$$
\begin{aligned}
&{\rm BR}(\bar{B} \to X_s\mu^+\mu^-)_{0.05<s<0.25} \\
&\quad = (1.46 \pm 0.11 \pm 0.10) \times 10^{-6},
\end{aligned}
\tag{15}
$$

where we have used ${\rm BR}(\bar{B} \to X_c e\nu_e) = 0.102$. The first uncertainty comes from the μ_b dependence and the second

[2] Complete results for the matrix elements, valid in the entire range of s, have been reported in [40] but are not yet publicly available.

[3] In this analysis we neglect the contribution of the real gluon bremsstrahlung calculated in [42] which changes the result by $\sim 1\%$.

[4] This conclusion has been reached in a discussion with M. Misiak.

one from $\Delta m_t^{\overline{\rm MS}}(m_t) = 5\,$GeV. To this one has to add the 6% uncertainty from the C factor and (conservatively) a $\sim 8\%$ uncertainty from the electromagnetic corrections. Adding all these uncertainties in quadratures we finally assign to the result the uncertainty of order $\pm 14\%$.

Integrating the differential rate (9) over the entire domain[5] $s_{\rm min} < s < s_{\rm max}$ where $s_{\rm min} = 4m_l^2/m_b^2$, $s_{\rm max} = (1 - m_s/m_b)^2$ one obtains the so-called "non-resonant" branching fraction which can be compared with the experimental data provided the contribution of the $\bar{c}c$ resonances is judiciously subtracted from the latter on the experimental side. Since the NNLO formulae for the matrix elements given in [39] are valid only for $s < 0.25$, following the prescription of [28] we have used for the region $s > 0.25$ only the formulae of [31] with $\mu_b = 2.5\,$GeV (because for $s < 0.25$ the formulae of [31] with $\mu_b = 2.5\,$GeV quite accurately reproduce the full NNLO results obtained with $\mu_b = 5\,$GeV) and assigned to the integral over this range of s the same μ_b uncertainty as has $\mathrm{d}\Gamma(\bar{B} \to X_s l^+ l^-)/\mathrm{d}s$ computed for $s = 0.25$. We get in this way

$$\mathrm{BR}(\bar{B} \to X_s \mu^+\mu^-)_{\rm non-res} = (4.39^{+0.24}_{-0.36} \pm 0.24) \times 10^{-6}\ , \qquad (16)$$

$$\mathrm{BR}(\bar{B} \to X_s e^+e^-)_{\rm non-res} = (7.26^{+0.25}_{-0.58} \pm 0.28) \times 10^{-6}\ , \qquad (17)$$

where the meaning of the errors is as previously. Taking into account the remaining uncertainties we estimate the total uncertainty of $\mathrm{BR}(\bar{B} \to X_s \mu^+\mu^-)_{\rm non-res}$ for ${}^{+13}_{-14}\%$ and of $\mathrm{BR}(\bar{B} \to X_s e^+e^-)_{\rm non-res}$ for ${}^{+11}_{-14}\%$. Our central values are in good agreement with the ones given in [28] but due to the normalization to the width of the semileptonic charmless decay the overall uncertainty is smaller even though we take into account the uncertainties related to the electromagnetic correction. Within the errors and uncertainties the SM prediction (16) is roughly in agreement with the published BELLE [43] and recent BaBar results, which together give [23] $\mathrm{BR}(\bar{B} \to X_s l^+ l^-)_{\rm non-res} = (6.2 \pm 1.7) \times 10^{-6}$, averaged over $l = \mu, e$, for the dilepton invariant mass $\sqrt{q^2} > 0.2\,$GeV. [6]

For a relatively clean comparison of the experimental measurements with the theoretical predictions of interest can also be used the rate integrated over the region of s above the $\bar{c}c$ resonances. We get there

$$\mathrm{BR}(\bar{B} \to X_s l^+ l^-)_{0.65<s<s_{\rm max}} = (2.32^{+0.17}_{-0.20} \pm 0.14) \times 10^{-7} \qquad (18)$$

($l = e$ or μ), where the first uncertainty, corresponding to the μ_b dependence, is estimated with the help of the prescription of [28] described above. A better estimate of this uncertainty will become possible once the calculation of [40] is available. However for this range of s the non-perturbative $1/m_b^2$ corrections of [37,38] constitute yet another potential source of uncertainty. For $s \gtrsim 0.8$ these corrections cannot be calculated reliably [38] ($s_m = 0.65$ of that paper corresponds to $s \approx 0.8$) which manifests itself in the negative values of the expression (9) for $s \to 1$. To estimate the uncertainty introduced by this factor we have computed $\mathrm{BR}(\bar{B} \to X_s l^+ l^-)_{0.65<s<s_{\rm max}}$ switching off the $1/m_b^2$ corrections in (9) for $s > 0.8$. At $\mu_b = 2.5\,$GeV this gives $\mathrm{BR}(\bar{B} \to X_s l^+ l^-)_{0.65<s<s_{\rm max}} = 2.66 \times 10^{-7}$. The difference of order 15% between this result and (18) can be interpreted as the uncertainty associated with the $1/m_b^2$ corrections. Adding all uncertainties in quadratures we finally assign to the result (18) an uncertainty of order $\pm 20\%$.

3 Scalar flavor changing neutral currents

Even in the MFV MSSM with $\tan\beta \gg 1$ ordinary one loop corrections involving charginos and stops can generate substantial FV couplings of neutral Higgs bosons to the down-type quarks ($q = s, d$) [3,8,7,12]. For sparticles sufficiently heavier than the charged Higgs boson (which sets the mass scale of the MSSM Higgs sector, as in the MSSM for $M_{H^+} \gtrsim 200\,$GeV $M_H \approx M_A \approx M_{H^+}$) the effects of these FV couplings can be described by the local Lagrangian of the form:

$$\mathcal{L}^{\rm eff} = -\bar{q}_{\rm L}\,[X_{\rm LR}]^{qb}\, b_{\rm R}(H^0 - \mathrm{i}A^0) - \bar{q}_{\rm R}\,[X_{\rm RL}]^{qb}\, b_{\rm L}(H^0 + \mathrm{i}A^0) + \mathrm{H.c.}\ , \qquad (19)$$

where in the so-called approximation of unbroken $SU(2) \times U(1)$ symmetry the amplitudes $[X_{\rm LR}]^{qb}$ are given by [20]

$$[X_{\rm LR}]^{qb} \approx - \frac{g_2^3}{4}\frac{m_b}{M_W}\left(\frac{m_t}{M_W}\right)^2 \frac{\tan^2\beta\; V^*_{tq} V_{tb}}{(1 + \tilde{\epsilon}_b \tan\beta)(1 + \epsilon_0 \tan\beta)}\ \epsilon_Y\ . \qquad (20)$$

The factors $\epsilon_Y \sim \mathcal{O}(1/16\pi^2)$, ϵ_0 and $\tilde{\epsilon}_b$ (see [20] for the analytical expressions) depend on the sparticle mass parameters; in particular, ϵ_Y is directly proportional to the mixing of left and right stops, that is to the parameter A_t [12]. The factors ϵ_0 and $\tilde{\epsilon}_b$ which depend on both, $\alpha_{\rm s}$ and the top Yukawa couplings, ensure proper resummation of the $(\tan\beta)$-enhanced terms from all orders of the perturbation expansion [8,7,12,14,20,21]. Their signs and magnitudes depend directly on the signs of the supersymmetric μ and A_t parameters. Generally, the resummation factors suppress the FV couplings for $\mu > 0$ [14] and enhance them for $\mu < 0$ [20]. The amplitudes $[X_{\rm RL}]^{qb}$ of the transitions $b_{\rm L} \to s_{\rm R}(d_{\rm R})$ are given by similar expressions but with m_b replaced by $m_{s(d)}$ and are, therefore, suppressed (but are, nevertheless, important for the B_s^0–$\bar{B}_s^0$ mixing [9,17,18]).

[5] Keeping $m_s \neq 0$ has numerically a very small impact on $\mathrm{d}\Gamma/\mathrm{d}s$ itself but $s_{\rm max} < 1$ for the upper integration limit partly cures the problem associated with the non-perturbative contributions to the differential rate, which for $s \to 1$ dominate in the expression (9) and make it negative in the vicinity of $s = 1$ [38].

[6] Our result for $\mathrm{BR}(\bar{B} \to X_s e^+e^-)_{\rm non-res}$ for $\sqrt{q^2} > 0.2\,$GeV is similar to (16): $(4.48^{+0.24}_{-0.37} \pm 0.24) \times 10^{-6}$. In the comparison with the BELLE result one also has to take into account the error in translating the "reduced" invariant mass $s = q^2/m_b^2$ into the experimental cut on the physical q^2.

The approximate formula (20) captures the main qualitative features of the FV couplings generated in the MFV MSSM. For more accurate estimates of their magnitude and dependences on the MSSM parameters one has to use, however, the more complicated approach developed in [20] which combines the resummation of the $(\tan\beta)$-enhanced terms with the complete diagrammatic one loop calculation. In principle, for $M_{\rm SUSY} \gg M_W$ one should also take into account that the couplings (19) are generated in the process of integrating out heavy sparticles at some scale $\mu_S \sim M_{\rm SUSY}$ and should be evolved down to the matching scale μ_0 using the RGEs similar to the RGEs for the quark Yukawa couplings in the SM

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\left[X_{\rm LR}\right]^{qb} = -8\frac{\alpha_{\rm s}}{4\pi}\left[X_{\rm LR}\right]^{qb} + \dots, \tag{21}$$

where we have retained only the effects of the QCD renormalization. As a result, the couplings $[X_{\rm LR}]^{qb}$ would be multiplied by the factor $[\alpha_{\rm s}(\mu_0)/\alpha_{\rm s}(\mu_S)]^{4/7}$, equal (for $\mu_0 = m_t$) to 1.073 for $\mu_S = 500$ and to 1.12 for $\mu_S = 1000$ GeV. To consistently take such effects into account one would have also to determine sparticle couplings at the scale μ_S (and use them to compute the amplitudes $[X_{\rm LR}]^{qb}$). Since for the correlations discussed in Sect. 4 only the values of $[X_{\rm LR}]^{qb}$ at μ_0 matter we will simply assume that sparticles are integrated out at the same scale $\mu_0 = m_t$.

With the FV couplings (20) the tree level exchanges of H^0 and A^0 generate at the scale μ_0 Wilson coefficients of the $\mathcal{O}^l_S$ and $\mathcal{O}^l_P$ operators

$$\begin{aligned} C^{l(1)}_S(\mu_0) &= -\frac{g_2^4}{8M_A^2}\frac{m_l m_b^{\overline{\rm MS}}(\mu_0)}{M_W^2} \\ &\times\left(\frac{m_t}{M_W}\right)^3 \frac{\tan^3\beta\, V^*_{tq}V_{tb}}{(1+\tilde\epsilon_b\tan\beta)(1+\epsilon_0\tan\beta)}\epsilon_Y \\ &\approx -C^{l(1)}_P(\mu_0)\ . \end{aligned} \tag{22}$$

Note that the expressions for $C^{l(1)}_S$ and $C^{l(1)}_P$ through their dependence (via ϵ_0 and $\tilde\epsilon_b$) on the coupling constants $\alpha_{\rm s}$ and $\alpha_t \equiv y_t^2/4\pi$ (where y_t is the top-quark Yukawa coupling) resum terms of order $\alpha_{\rm s}^n\alpha_t^m\tan^{n+m}\beta$ $(n,m \ge 0)$ from all orders of perturbation theory. Since the operators $m_b\mathcal{O}_{S,P}$ are renormalization scale invariant with respect to the strong interactions, the QCD evolution of

$$\tilde C^l_{S,P} \equiv C^{l(1)}_{S,P} + \frac{\alpha_{\rm s}}{4\pi}C^{l(2)}_{S,P} + \cdots \tag{23}$$

reduces to the multiplication of $\tilde C^l_{S,P}(\mu_0)$ by the factor $[m_b(\mu_b)^{\overline{\rm MS}}/m_b^{\overline{\rm MS}}(\mu_0)]$. If as in (22) $C^{l(1)}_{S,P}(\mu_0) \propto m_b^{\overline{\rm MS}}(\mu_0)$, the dependence on $m_b^{\overline{\rm MS}}$ of the formula for $\mathrm{BR}(B^0_q \to l^+l^-)$ cancels against the factor $1/m_b^{\overline{\rm MS}}(\mu_b)$ present in the matrix element of the $\mathcal{O}_{S,P}$ operators [46]:

$$\begin{aligned} \langle 0|\bar q_{\rm L} b_{\rm R}(\mu_b)|\bar B^0_q\rangle &= \mathrm{i}\hat F_{B_q}\frac{M^2_{B^0_q}}{m_b^{\overline{\rm MS}}(\mu_b)+m_q^{\overline{\rm MS}}(\mu_b)} \\ &\approx \mathrm{i}\hat F_{B_q}\frac{M^2_{B^0_q}}{m_b^{\overline{\rm MS}}(\mu_b)}. \end{aligned} \tag{24}$$

The complete $\mathcal{O}(\alpha_{\rm s})$ calculation of the scalar operators contribution to $\mathrm{BR}(B^0_q \to l^+l^-)$ in the MSSM would therefore require only computing higher order corrections to the matching conditions at the scale μ_0, that is to resum all contributions to $C^{l(2)}_S$ and $C^{l(2)}_P$ of order $\alpha_{\rm s}(\alpha_{\rm s}^n\alpha_t^m\tan^{n+m}\beta)$ for $n, m \ge 0$.

One can also take a more general point of view and assume that the scalar operators $\mathcal{O}^l_{S,P}$ are generated at the scale μ_0 by some yet unknown physics and investigate their effects on the $b \to sl^+l^-$ and $b \to dl^+l^-$ transitions without any reference to the more fundamental theory, treating the Wilson coefficients $\tilde C^l_{S,P}$ as free parameters. Assuming dominance of the scalar $\mathcal{O}^l_{S,P}$ operators, the formula for $\Gamma(B^0_q \to l^+l^-)$ [12,13] takes the form

$$\begin{aligned} &\Gamma(B^0_q \to l^+l^-) \\ &\approx M_{B^0_q}\frac{\left(G_{\rm F}\alpha_{\rm em}M_{B^0_q}\hat F_{B_q}\right)^2}{64\pi^3}\left(\frac{M_{B^0_q}}{m_b^{\overline{\rm MS}}(\mu_b)}\right)^2 \\ &\times |V^*_{tq}V_{tb}|^2\left\{\left|\tilde C^l_S(\mu_b)\right|^2 + \left|\tilde C^l_P(\mu_b)\right|^2\right\}, \end{aligned}$$

that is

$$\begin{aligned} &\mathrm{BR}(B^0_s \to l^+l^-) \approx 4.27\times 10^{-7} \\ &\times\left(\frac{\hat F_{B_s}}{238\,\mathrm{MeV}}\right)^2\left|\frac{V^*_{ts}V_{tb}}{0.04}\right|^2\left(\frac{4.2\,\mathrm{GeV}}{m_b^{\overline{\rm MS}}(\mu_b)}\right)^2 \\ &\times\frac{1}{2}\left\{\left|\tilde C^l_S(\mu_b)\right|^2 + \left|\tilde C^l_P(\mu_b)\right|^2\right\}. \end{aligned} \tag{25}$$

The recent CDF upper limit [23] $\mathrm{BR}(B^0_s \to \mu^+\mu^-) < 0.95\times 10^{-6}$ at 90% C.L. sets therefore the stringent bound [13]

$$\begin{aligned} &\frac{1}{2}\left\{\left|\tilde C^\mu_S(\mu_b)\right|^2 + \left|\tilde C^\mu_P(\mu_b)\right|^2\right\} \\ &\lesssim 2.2\times\left(\frac{238\,\mathrm{MeV}}{\hat F_{B_s}}\right)^2\left(\frac{m_b^{\overline{\rm MS}}(\mu_b)}{4.2\ \mathrm{GeV}}\right)^2. \end{aligned} \tag{26}$$

A similar bound can also be derived for $\tilde C^e_{S,P}(\mu_b)$ by using the corresponding experimental upper limit $\mathrm{BR}(B^0_s \to e^+e^-) < 5.4\times 10^{-5}$ [45] but it is two orders of magnitude weaker. Analogous bounds on the (universal under the assumption of MFV) Wilson coefficients $\tilde C^{e,\mu}_{S,P}(\mu_b)$ that can be derived from the experimental upper limits $\mathrm{BR}(B^0_d \to \mu^+\mu^-) < 1.6\times 10^{-7}$ [35,23] and $\mathrm{BR}(B^0_d \to e^+e^-) < 8.3\times 10^{-7}$ [45] are less interesting as they depend on the value of $|V_{td}|$, the determination of which can also be affected by the new physics that gives rise to the scalar operators [18].

As follows from the formula (22), in the MSSM $\tilde C^l_{S,P} \approx C^{l(1)}_{S,P} \propto m_l$, so that the effects of the scalar operators can be measurable only for the $\mu^+\mu^-$ and $\tau^+\tau^-$ channels (the latter being very difficult for experimental searches so that at present no limit on $\mathrm{BR}(B^0_s \to \tau^+\tau^-)$ is available). For $\tan\beta \sim 40$–50, substantial stop mixing and $\mu < 0$, when the resummation of the leading $\tan^n\beta$ terms enhances the FV violating couplings, $|C_S^{\mu(1)}| \approx |C_P^{\mu(1)}|$ could be as large as ~ 10 leading to $\mathrm{BR}(B^0_s \to \mu^+\mu^-) \sim 10^{-5}$ [12, 20]. The bound (26) eliminates therefore a large portion of the general MSSM parameter space. Moreover, as has been demonstrated in [17,20], in such cases also the contribution of the FV couplings of the neutral MSSM Higgs bosons to the B^0_s–$\bar{B}^0_s$ mass difference ΔM_s is large and the experimental limit $(\Delta M_s)^{\mathrm{exp}} \gtrsim 14/\mathrm{ps}$ [26] becomes in most cases more constraining (see the next section).

It should be stressed, however, that the bounds like (26) are completely independent of the specific way of generation of the coefficients $|\tilde{C}_{S,P}|$ and are valid generally, and not only in supersymmetry.[7] In particular, one can imagine that the operators $\mathcal{O}^l_{S,P}$ are not due to the neutral Higgs boson exchanges between the FV violating down-type quark vertices and the leptonic vertices in which case sizable effects of the scalar operators $\mathcal{O}^l_{S,P}$ could be present in any of the $b \to s(d)l^+l^-$ transitions (for any lepton) and not accompanied by large contributions to the B^0_s–$\bar{B}^0_s$ mixing amplitude as in the MSSM. For this reason, the remaining analysis of this section will be done in a general framework. We will return to the MSSM only in the next section.

The general bound (26) on $|\tilde{C}^\mu_{S,P}|$ allows for an immediate estimate of the impact the scalar operators $\mathcal{O}^\mu_{S,P}$ may have on the rate of the inclusive process $B \to X_s\mu^+\mu^-$. Similar estimates can also be made for the $B \to X_s e^+e^-$ and $B \to X_s\tau^+\tau^-$ processes. From the formula (9) for the contribution of the scalar operators to the differential rate we get [15,29]

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}s}\Delta\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-) \\
&\approx \frac{\mathrm{BR}(\bar{B} \to X_c e\nu_e)}{\left(1 - \frac{2\alpha_s(m_b)}{3\pi}h(0)\right)}\frac{1}{C}\left|\frac{V^*_{ts}V_{tb}}{V_{cb}}\right|^2\left(\frac{\alpha_{\mathrm{em}}}{2\pi}\right)^2 \\
&\times (1-s)^2\left\{\frac{3}{2}s\left|\tilde{C}^\mu_S(\mu_b)\right|^2 + \frac{3}{2}s\left|\tilde{C}^\mu_P(\mu_b)\right|^2\right. \\
&\left. + 6\frac{m_\mu}{m_b}\tilde{C}^\mu_P(\mu_b)C^{\mathrm{eff}}_{10}(s,\mu_b)\right\}, \qquad (27)
\end{aligned}$$

where we have used the normalization to the width of the semileptonic charmless decays and for simplicity dropped the non-perturbative correction factor appearing in the denominators of the formula (13). As remarked below the formulae (12), the contribution of the operators $\mathcal{O}^l_{S,P}$ to the inclusive rate $\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-)$ depends on the choice of the renormalization scale μ_b. Since following [41] we use $m^{1S}_b = 4.69\,\mathrm{GeV}$ leading to $m^{\overline{\mathrm{MS}}}_b(m^{\overline{\mathrm{MS}}}_b) \approx 4.2\,\mathrm{GeV}$ for the value of the running b-quark mass, in what follows we will treat as free parameters the Wilson coefficients $\tilde{C}^\mu_{S,P}$ taken at $\mu_b = 4.2\,\mathrm{GeV}$. The uncertainty related to the variation of the scale $\mu_b \to \mu'_b$ in the formula (27) is then roughly (ascribing for the estimation purpose to the interference term the same μ_b dependence as have the other two terms) given by $[m^{\overline{\mathrm{MS}}}_b(\mu'_b)/m^{\overline{\mathrm{MS}}}_b(\mu_b)]^2$ and is estimated to be $^{+22}_{-25}\%$. This uncertainty has to be, of course, combined with the ones stemming from unknown electromagnetic corrections and the C-factor (14). Inserting numbers in the formula (27) we get

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}s}\Delta\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-) \approx 4.7 \times 10^{-7} \\
&\times (1-s)^2\left\{s\left|\tilde{C}^\mu_S(\mu_b)\right|^2 + s\left|\tilde{C}^\mu_P(\mu_b)\right|^2\right. \\
&\left. + 4\frac{m_\mu}{m_b}\tilde{C}^\mu_P(\mu_b)C^{(1)}_{10}\right\}. \qquad (28)
\end{aligned}$$

Integrating over the full $(0,1)$ range of s and taking into account the limit (26) with $m^{\overline{\mathrm{MS}}}_b(\mu_b) = 4.2\,\mathrm{GeV}$ for $\mu_b = 4.2\,\mathrm{GeV}$ we obtain the estimate of the maximal possible contribution of the scalar operators to the "non-resonant" branching ratio:

$$\begin{aligned}
&\Delta\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-)_{\mathrm{non-res}} \\
&\lesssim 1.7 \times f \times \left(1 \pm 0.5 \times \sqrt{r/f}\right) \times 10^{-7}, \qquad (29)
\end{aligned}$$

where

$$f \equiv \left(\frac{238\,\mathrm{MeV}}{\hat{F}_{B_s}}\right)^2, \qquad 0.78 < f < 1.32, \qquad (30)$$

and the factor

$$0 \le r \le 2 \qquad (31)$$

depends on the relative magnitudes of $|\tilde{C}^\mu_P|$ and $|\tilde{C}^\mu_S|$: $r = 0$ for $|\tilde{C}^\mu_P| = 0$ and $r = 2$ for $|\tilde{C}^\mu_S| = 0$; for $|\tilde{C}^\mu_P| = |\tilde{C}^\mu_S|$, as in the MSSM, $r = 1$. The $\pm$ refers to the two possible signs of the interference term depending on the sign of $\tilde{C}^\mu_P$ (the interference is constructive for $\tilde{C}^\mu_P < 0$). We have used the approximate SM value $\tilde{C}^{\mathrm{eff}}_{10}(s,\mu_b) \approx C^{(1)}_{10} \approx -4.2$ and $m_b = m^{\mathrm{pole}}_b = 4.8\,\mathrm{GeV}$ in the interference term. Thus, the maximal effect of the scalar operator is 3.7×10^{-7} for $f = 1.32$ and $r = 2$ (2.55×10^{-7} for $f = r = 1$). Comparing with the SM result (16) we conclude that the maximal contribution of the scalar operators allowed by the CDF limit (1) is at most at the level of 8% for $f = 1.32$,

[7] The bound (26) is valid also if the new physics, which gives rise to non-zero $\tilde{C}^l_{S,P}$ involves sources of FV other than the CKM matrix, provided the coefficients $\tilde{C}^l_{S,P}$ are (superficially) normalized as in (2). More generally, for a given lepton pair l^+l^- the experimental upper limits on $\mathrm{BR}(B^0_s \to l^+l^-)$ and $\mathrm{BR}(B^0_d \to l^+l^-)$ set then independent bounds on the products (assuming that the Wilson coefficients are still normalized as in (2)) $|V^*_{tq}V_{tb}|^2\left\{\left|\tilde{C}^l_S\right|^2 + \left|\tilde{C}^l_P\right|^2\right\}$ for $q = s$ and $q = d$, respectively, which can be directly used to constrain the maximal possible effects of the scalar operators in inclusive or exclusive $\bar{B} \to X_s l^+l^-$ and $\bar{B} \to X_d l^+l^-$ decays.

$r = 2$ (5% for $f = r = 1$), that is, substantially smaller than the estimated uncertainty of the SM prediction. This is in sharp contrast with the findings of [29], where it has been claimed that even within the so-called minimal SUGRA framework the ratio $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)/\mathrm{BR}(B_s^0 \to e^+e^-)$ can reach values as large as 2–3, corresponding to the contribution of the scalar operators as large as 100–200%.

For the branching ratio integrated over the range $0.05 < s < 0.25$ we find

$$\begin{aligned} &\Delta\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-)_{0.05<s<0.25} \\ &\lesssim 0.43 \times f \times \left(1 \pm 0.88 \times \sqrt{r/f}\right) \times 10^{-7}, \end{aligned} \tag{32}$$

that is, the maximal effect is again of order 8% for $f = 1.32$, $r = 2$ (5.5% for $f = r = 1$), much smaller than the estimated uncertainty of the SM prediction for this range. For the range of s above the $\bar{c}c$ resonances the limit (1) implies

$$\begin{aligned} &\Delta\mathrm{BR}(B \to X_s\mu^+\mu^-)_{0.65<s<1} \\ &\lesssim 0.22 \times f \times \left(1 \pm 0.16 \times \sqrt{r/f}\right) \times 10^{-7}. \end{aligned} \tag{33}$$

For this s range the maximal possible contribution of the scalar operators increases the branching fraction by $\sim 15\%$ for $f = 1.32$, $r = 2$ (11% for $f = r = 1$); that is, again the effects of the scalar operators are not greater than the estimated uncertainty of the SM prediction.[8] Estimates of $\Delta\mathrm{BR}(B \to X_s e^+e^-)$ can also be obtained in a similar manner.

Experimentally first measured were the exclusive B decay modes $\bar{B} \to Kl^+l^-$ and $\bar{B} \to K^*l^+l^-$ [44]. For $\bar{B} \to Kl^+l^-$, which will be of interest for us here,[9] the recent results for the "non-resonant" rates are [23]: $\mathrm{BR}(\bar{B} \to K\mu^+\mu^-) = (4.8^{+1.5}_{-1.3} \pm 0.3 \pm 0.1) \times 10^{-7}$ and $\mathrm{BR}(\bar{B} \to Kl^+l^-) = (4.8^{+1.0}_{-0.9} \pm 0.3 \pm 0.1) \times 10^{-7}$ averaged over e and μ (BELLE) and $\mathrm{BR}(\bar{B} \to K\mu^+\mu^-) = (4.8^{+2.5}_{-2.0} \pm 0.4) \times 10^{-7}$ and $\mathrm{BR}(\bar{B} \to Kl^+l^-) = (6.9^{+1.5}_{-1.3} \pm 0.6) \times 10^{-7}$ (BaBar). The main uncertainty of the theoretical $\mathrm{BR}(\bar{B} \to Kl^+l^-)$ calculation is related to the determination of the non-perturbative matrix elements of the relevant operators between the initial and final meson states. Different techniques used for this purpose resulted in the SM predictions for this branching fraction spanning the range (3.0–6.9)$\times 10^{-7}$ [47,48,28]. Within the experimental errors the new experimental results are in fair agreement with the SM-based NNLO theoretical estimate given by Ali et al. [28]: $\mathrm{BR}(\bar{B} \to Kl^+l^-)_{\mathrm{non-res}} = (3.5 \pm 1.2) \times 10^{-7}$. Substantial lowering of the SM prediction compared to the earlier one of Ali et al. (based on the NLO calculation) [48], $\mathrm{BR}(\bar{B} \to Kl^+l^-)_{\mathrm{non-res}} = (5.7 \pm 1.2) \times 10^{-7}$ was mainly due to the superficial lowering of values of the form factors parameterizing the operator matrix elements. This was motivated by the fact that the $q^2 = 0$ value of the $T_1(q^2)$ form factor obtained using the so-called QCD light cone sum rules (LCSR) gave, compared to the data, too high a branching fraction for the $\bar{B} \to K^*\gamma$ mode [49], suggesting that the LCSR method systematically overestimates the form factors.

The contribution of the scalar operators to the branching fraction $\mathrm{BR}(\bar{B} \to K\mu^+\mu^-)_{\mathrm{non-res}}$ has been analyzed in [13]. At that time only the upper limit $\mathrm{BR}(B^+ \to K^+\mu^+\mu^-) < 5.2 \times 10^{-6}$ was available [27], so the conclusion of [13] was that the constraint imposed on $|\tilde{C}_S^\mu|^2 + |\tilde{C}_P^\mu|^2$ by the limit $\mathrm{BR}(B_s^0 \to \mu^+\mu^-) < 2.6 \times 10^{-6}$ was significantly stronger than the one that could be obtained from the limit on $\mathrm{BR}(\bar{B} \to K^+\mu^+\mu^-)$. With the new numbers the situation is somewhat different and we summarize it below.

The scalar operators contribution to the non-resonant branching ratio can be written as [13]

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}q^2}\Delta\mathrm{BR}(\bar{B} \to Kl^+l^-)_{\mathrm{non-res}} \\ &= \frac{\tau_B}{\pi}\left(\frac{G_{\mathrm{F}}\alpha_{\mathrm{em}}}{16\pi^2}\right)^2 \frac{|V_{ts}^*V_{tb}|^2}{M_B^3}\lambda^{1/2}(q^2, M_B^2, M_K^2)\beta_l(q^2) \\ &\times \left\{q^2\beta_l^2(q^2)|\delta F_S|^2 + q^2|\delta F_P|^2 + 2q^2\mathrm{Re}(F_P^*\delta F_P)\right. \\ &\left.+ 2m_l(M_B^2 - M_K^2 + q^2)\mathrm{Re}(F_A^*\delta F_P)\right\}, \end{aligned} \tag{34}$$

where q^2 is the physical lepton pair invariant mass, $\beta_l(q^2) = \sqrt{1 - 4m_l^2/q^2}$ and

$$\begin{aligned} \delta F_{S,P} &= \frac{1}{2}\frac{C_{S,P}^l(\mu_b)}{m_b^{\overline{\mathrm{MS}}}(\mu_b)}(M_B^2 - M_K^2)\ f_0(q^2), \\ F_A &= C_{10}^{\mathrm{eff}} f_+(q^2), \\ F_P &= m_l C_{10}^{\mathrm{eff}} \\ &\times \left\{\frac{M_B^2 - M_K^2}{q^2}\left[f_+(q^2) - f_0(q^2)\right] - f_+(q^2)\right\}. \end{aligned} \tag{35}$$

The coefficient C_{10}^{eff} differs from $\tilde{C}_{10}^{\mathrm{eff}}(s, \mu_b)$ given in (12) by setting to zero the functions $\omega_9(s)$ (the effects of $\omega_9(s)$ are supposed to be taken into account in the form factors $f_0(q^2)$ and $f_+(q^2)$). Note that C_{10}^{eff} [31], and hence the whole formula (34), is independent of the renormalization scale μ_b. Following the recipe of [28] for the central values of the form factors $f_0(q^2)$ and $f_+(q^2)$, as well as for $f_T(q^2)$ appearing below, in (39), we use their lowest values obtained within the LCSR approach which amounts to using the formula (3.7) of [48] with the parameters collected in Table V of that paper. At the same time, again following [28], we ascribe to the values of the form factors the uncertainty of order 15%. The form factors introduce therefore in the results for $(\mathrm{d}/\mathrm{d}q^2)\Delta\mathrm{BR}(\bar{B} \to Kl^+l^-)_{\mathrm{non-res}}$ the largest (barring the discussion how big errors are introduced by using the effective Lagrangian with non-local coefficients $C_9^{\mathrm{eff}}(q^2)$, $C_7^{\mathrm{eff}}(q^2)$, for the exclusive process) uncertainty of order 30%.

Integrating over q^2 in the kinematical limits $4m_\mu^2 < q^2 < (M_B - M_K)^2$ and assuming that the new physics

[8] With the old limit $BT(B_s^0 \to \mu^+\mu^-) < 2 \times 10^{-6}$ [24] the effects of the scalar operators in this range of s could be almost twice as big as the estimated uncertainty.

[9] As analyzed in [13], the contribution of the scalar operators to $\mathrm{BR}(\bar{B} \to K^*\mu^+\mu^-)$ is too small to be interesting.

contribution to Wilson coefficients other than $\tilde{C}^{\mu}_{S,P}$ is negligible we obtain for the dimuon mode

$$\Delta\mathrm{BR}(\bar{B} \to K\mu^+\mu^-)_{\mathrm{non-res}} \approx 6.36 \times 10^{-8} \times \left\{ a \left(\left|\tilde{C}^{\mu}_{S}\right|^2 + \left|\tilde{C}^{\mu}_{P}\right|^2 \right) - b\, \tilde{C}^{\mu}_{P} \right\}, \quad (36)$$

where for $\mu_b = 4.2\,\mathrm{GeV}$ $a = 0.30 \pm 0.10$ and $b = 0.19 \pm 0.06$. The uncertainties of a and b are due to the uncertainties of the form factors $f_0(q^2)$ and $f_+(q^2)$. Through the form factors the total uncertainty of the scalar operators contribution is obviously strongly correlated with the uncertainty of the SM prediction. Sticking to the central values of a and b and taking maximal values of $|\tilde{C}^{\mu}_{S}|$ and $|\tilde{C}^{\mu}_{P}|$ allowed by the bound (26) we get

$$\Delta\mathrm{BR}(\bar{B} \to K\mu^+\mu^-)_{\mathrm{non-res}} \lesssim 0.8 \times f \times \left(1 \pm 0.45\sqrt{r/f}\right) \times 10^{-7}, \quad (37)$$

that is, the maximal possible contribution of the scalar operators to the non-resonant branching fraction can be (for $\tilde{C}^{\mu}_{P} < 0$, $\tilde{C}^{\mu}_{S} = 0$, and the lowest possible value of $\hat{F}_{B_s}$, i.e. for the + sign, $r = 2$, $f = 1.32$) as large as 1.7×10^{-7}, roughly of the same magnitude as the error of the experimental result and 1.5 times bigger than the estimated [48,13,28] uncertainty ($\sim 1.2 \times 10^{-7}$) of the SM prediction. Similar estimates can also be done for $\Delta\mathrm{BR}(\bar{B} \to K e^+e^-)_{\mathrm{non-res}}$.

Finally, an experimentally interesting quantity [13,50] may be the integrated over q^2 forward–backward lepton asymmetry measured in this decay given by[10]

$$A_{\mathrm{FB}} = \frac{\tau_B}{\mathrm{BR}(\bar{B} \to K\mu^+\mu^-)} \left(\frac{G_{\mathrm{F}}\alpha_{\mathrm{em}}}{16\pi^2}\right)^2 \frac{|V^*_{ts}V_{tb}|^2}{\pi M_B^3} \times \int \mathrm{d}q^2\, m_l \lambda(q^2, M_B^2, M_K^2)\, \beta_l^2(q^2)\, \mathrm{Re}(F_V^* \delta F_S), \quad (38)$$

where

$$F_V = C_9^{\mathrm{eff}}(\mu_b) f_+(q^2) + 2m_b C_7^{\mathrm{eff}}(\mu_b) \frac{f_T(q^2)}{M_B + M_K}, \quad (39)$$

with C_9^{eff} and C_7^{eff} differing from $\tilde{C}_9^{\mathrm{eff}}$ and $\tilde{C}_7^{\mathrm{eff}}$ of (12) by setting to zero[11] the functions $\omega_9(s)$ and $\omega_7(s)$. The asymmetry A_{FB} vanishes in the SM in which $F_S = \delta F_S = 0$. For the dimuon channel, integrating over the whole q^2 range and using $\mu_b = 4.2\,\mathrm{GeV}$ we get

$$A_{\mathrm{FB}} \approx \frac{1}{\mathrm{BR}(\bar{B} \to K\mu^+\mu^-)} \times (4.9 \pm 1.6) \times \tilde{C}^{\mu}_{S} \times 10^{-9} \lesssim \pm \left[\frac{4.8 \times 10^{-7}}{\mathrm{BR}(\bar{B} \to K\mu^+\mu^-)}\right] \times (1.5 \pm 0.5) \times \sqrt{r' f}\% \quad (40)$$

where $0 < r' < 2$ ($r' = 0$ for $C^{\mu}_{S} = 0$ and $r' = 2$ for $C^{\mu}_{P} = 0$; for $|\tilde{C}^{\mu}_{S}| = |\tilde{C}^{\mu}_{P}|$, as in the MSSM with, $r' = 1$). The uncertainty of this result being dominated by the 30% uncertainty, arising from the form factors $f_+(q^2)$ and $f_T(q^2)$), is of course strongly correlated with the uncertainty of the total branching ratio. Still, the maximal possible asymmetry allowed by the limit (26) is of the order of a percent and may be detectable in the future.

We conclude that given the experimental limit (1), the effects of the scalar operators in the inclusive process are typically of order 5–15%, always smaller than the estimated uncertainty of the SM NNLO prediction. On the other hand, the maximal allowed contribution of the scalar operators to $\mathrm{BR}(\bar{B} \to K\mu^+\mu^-)$, although larger than the estimates of the theoretical uncertainty of the SM prediction made in [48,13,28], is only roughly of the order of the present experimental error. While the latter can shrink in the near future, the spread of the different SM-based theoretical predictions and the problems with the form factor values obtained using the QCD LCSR may suggest that the true uncertainty of the SM prediction is larger than estimated in [48,13,28], thus preventing a reliable comparison of the theoretical predictions with the data. The forward–backward asymmetry of the muon distribution, if detected in the high statistic data, could also be indicative of the scalar operators contribution (the asymmetry vanishes if only the SM operators contribute) but its translation into the values of $\tilde{C}^{\mu}_{S}$ and $\tilde{C}^{\mu}_{P}$ depends on the form factors too. Thus, before the status of the form factors is clarified and the errors associated with them reliably estimated the exclusive mode $\bar{B} \to K\mu^+\mu^-$, although potentially interesting, will not be able to put constraints on the coefficients $\tilde{C}^{\mu}_{S}$ and $\tilde{C}^{\mu}_{P}$.

The coefficients $\tilde{C}^{\mu}_{S}$ and $\tilde{C}^{\mu}_{P}$ of the most interesting (largest allowed) magnitude cannot be, however, as in supersymmetry, due to the tree level exchanges of the neutral Higgs bosons between the effective quark FV vertices and the Higgs–lepton–lepton vertices. As we shall see on the MSSM example in the next section, in such a case possible effects of the scalar operators (apart from being slightly reduced by the relation $|C^{l(1)}_{S}| \approx |C^{l(1)}_{P}|$ so that $r = r' = 1$) can be further constrained by the $\bar{B}^0_s$–B^0_s mixing.

4 Correlation with $\mathrm{BR}(\bar{B} \to X_s\gamma)$ and the $\bar{B}^0_s$–B^0_s mass difference

In assessing potential effects of the scalar operators in the preceding section we have ignored the fact that the new physics, which gives rise to them, can also modify the remaining Wilson coefficients. In the MFV MSSM charginos and stops as well as the charged Higgs boson H^+ contribute to $C_{10}(\mu_0)$ and $C_9(\mu_0)$ through the box, Z^0-penguin and, in the case of $C_9(\mu_0)$, also through the photonic penguin diagrams. Likewise the coefficients of the C_7 and C_8 are modified by loops containing these particles. It should also be stressed that supersymmetric contributions

[10] Of interest can also be unintegrated differential asymmetry [51].

[11] For $q^2/m_b^2 > 0.25$ we also set to zero the functions $F_i^{(7,9)}$.

to $C_X^{(2)}(\mu_0)$ in (4) necessary for complete NNLO calculations are only partly known for C_7 and C_8 (and only for a scenario with light right-handed stop and charginos) [52] and are unknown for the other coefficients in (2). Out of the Wilson coefficients relevant for the $b \to sl^+l^-$ transition only the modulus of $\tilde{C}_7^{\rm eff}$ (but not its sign) is rather well constrained by the measurement of BR($\bar{B} \to X_s\gamma$). The other coefficients can still accommodate substantial new physics contributions.

If H^+ is light – a necessary condition for generating in the MSSM nonegligible Wilson coefficients of the scalar operators – its contribution to $\tilde{C}_7^{\rm eff}$ is substantial and has the same sign as the SM contribution. Therefore it must be cancelled by the chargino-stop contribution. For $\tan\beta \gg 1$ the latter is proportional to $\tan\beta$ and can be very large if these particles are light. Its sign depends on the sign of $A_t\mu$ and for $A_t\mu > 0$ (in our phase convention) it is opposite to the sign of the W^-t and H^+t loops so that the cancellation is indeed possible. Since for $A_t\mu > 0$ the Wilson coefficient $C_P^{l(1)}$ is negative ($C_S^{l(1)}$ is positive), the requirement that the calculated in the MSSM BR($\bar{B} \to X_s\gamma$) agrees for a light H^+ with the experimental result necessarily leads to a positive contribution of the interference term Re($\tilde{C}_{10}^{\rm eff}C_P^{l(1)*}$) in the formulae (9) and (27) (recall that the SM contribution to $\tilde{C}_{10}^{(1)}$ is also negative) so that in the estimates (29), (32), (33), (37) and (40) the + signs apply.

In principle the chargino-stop contribution could even reverse the sign of $\tilde{C}_7^{\rm eff}$ leading to a value of BR($\bar{B} \to X_s\gamma$) compatible with the experimental result. The sign of the Re($\tilde{C}_7^{\rm eff}\tilde{C}_9^{\rm eff*}$) term in the formula (9) would then be changed modifying predictions for the $\bar{B} \to X_sl^+l^-$ rates. Such a situation, which could most easily be distinguished by measuring the dilepton invariant mass spectrum and the forward–backward asymmetry in the BR($\bar{B} \to X_sl^+l^-$) [53], requires light, ~ 100 GeV, charginos and stops and, for light H^+ and $\tan\beta \gg 1$, is strongly fine tuned [54]. Much more natural appears the possibility that charginos and stops are rather heavy and their contribution to $\tilde{C}_7^{\rm eff}$, despite substantial stop mixing necessary for generating a large $C_{S,P}^{l(1)}$, is small, just of the right magnitude (and sign) to cancel the contribution of the charged Higgs boson. In such a scenario the value of $\tilde{C}_7^{\rm eff}$ must be close to the one predicted in the SM and the contributions of stops and charginos to $C_{10}^{(1)}(\mu_0)$ and $C_9^{(1)}(\mu_0)$ is, as we have checked by using the formulae of [55], negligible.

The H^+ contribution to $C_{10}^{(1)}(\mu_0)$ and $C_9^{(1)}(\mu_0)$ through the box diagrams, Z^0 and photonic penguins has been computed in [55]. For $\tan\beta \gg 1$ these contributions are not enhanced and are negligible even for the charged Higgs boson mass as low as 200 GeV. As has been found in [56,12] the H^+t loops also generate the FV couplings (19) and the resulting contribution to $C_{S,P}^{l(1)}$ grows as $\tan^2\beta$. However, for $M_{H^+} \gtrsim 200$ GeV and $\tan\beta \lesssim 50$ this contribution to the coefficients $C_{S,P}^{\mu(1)}$ are roughly two orders of magnitude below the upper limit (26) and, hence, their impact on the $\bar{B} \to X_sl^+l^-$ rate can also be neglected.

Thus, for sparticles heavier than, say, 500 GeV, the only sizable SUSY effects in the $b \to s\mu^+\mu^-$ transitions can be due to scalar operators.

As has been observed in [9,17,20], in the MFV MSSM whenever the coupling $[X_{\rm LR}]^{sb}$ (20) is large, the tree level exchanges of the neutral Higgs bosons H^0 and A^0 between the tree level effective vertices (19) give also a large negative contribution to the mass difference ΔM_s of the $\bar{B}_s^0$ and B_s^0 mesons. In the so-called approximation of unbroken $SU(2) \times U(1)$ symmetry, in which also the formula (20) is valid, one gets [20]

$$\delta(\Delta M_s) = -\frac{12.8}{\rm ps}\left[\frac{\tan\beta}{50}\right]^4 \times \left[\frac{\hat{F}_{B_s}}{238\,{\rm MeV}}\right]^2 \left[\frac{|V_{ts}|}{0.04}\right]^2 \left[\frac{m_b(\mu_0)}{3\,{\rm GeV}}\right] \left[\frac{m_s(\mu_0)}{60\,{\rm MeV}}\right] \times \left[\frac{m_t^4}{M_W^2M_A^2}\right] \left[\frac{16\pi^2\epsilon_Y}{(1+\epsilon_0\tan\beta)(1+\tilde{\epsilon}_b\tan\beta)}\right]^2 \quad (41)$$

(the analogous contribution to the $\bar{B}_d^0$–B_d^0 mass difference, being suppressed by the ratio m_d/m_s, is negligible). Typically the couplings $[X_{\rm LR}]^{sb}$ which give rise to $C_{S,P}^{\mu(1)}$ saturating the bound (26) lead to ΔM_s below the present lower experimental limit ~ 14/ps [26].

In order to see how the possible effects of the scalar operators in the $b \to s\mu^+\mu^-$ transitions are limited by the experimental lower bound on the $\bar{B}_s^0$–B_s^0 mass difference ΔM_s we present in Figs. 1a–f scatter plots of the combination $(1/2)\left(|C_S^{\mu(1)}(\mu_b)|^2 + |C_P^{\mu(1)}(\mu_b)|^2\right) \approx |C_S^{\mu(1)}(\mu_b)|^2 \approx |C_P^{\mu(1)}(\mu_b)|^2$ for $\mu_b = 4.2$ GeV versus ΔM_s calculated using the approach developed in [20] for a few combinations of the parameters $(M_A, \tan\beta)$.[12] The plots have been obtained by scanning over the MFV MSSM parameters (in the sense explained in more detail in [20]) with the lower bound on sparticle masses $M_{\rm SUSY} \gtrsim 500$ GeV. More specifically, we have scanned the relevant parameters in the ranges such that 500 GeV $< m_{C_1} <$ 1 TeV, with $0.75 < |M_2/\mu| < 1.5$, $1 < m_{\tilde{g}}/M_2 < 3$; $0.7 < M_{\tilde{t}_1}/m_{C_1} < 1.3$, $1.1 < M_{\tilde{t}_2}/M_{\tilde{t}_1} < 1.7$ and $-35° < \theta_{\tilde{t}} < 35°$; $0.5 < M_{\tilde{b}_{\rm R}}/m_{\tilde{g}} < 0.9$, with $A_b = A_t$; masses of the first two generations have been taken as $\max(M_{\tilde{b}_{\rm R}}, M_{\tilde{t}_{\rm L}})$. All points for which the computed BR($\bar{B} \to X_s\gamma$) does not agree with the experimental result have been rejected. We have used $|V_{ts}| = 0.04$ and $\hat{F}_{B_s} = 238$ MeV, but the limits for other values of these parameters can be obtained by simple rescalings. Horizontal lines in Figs. 1a–f show the upper bound (26) on $(1/2)\left(|\tilde{C}_S^{\mu}|^2 + |\tilde{C}_P^{\mu}|^2\right)$ for $\mu_b = 4.2$ GeV

[12] In producing these plots we have corrected a bug in our Fortran code which resulted in using in [17,18,20] $C_{S,P}^{\mu(1)}(\mu_0)$ instead of $C_{S,P}^{\mu(1)}(\mu_b)$ in calculating BR($B_{s,d}^0 \to \mu^+\mu^-$). As a result numerical values of this ratios in the figures of these references should be rescaled upwards roughly by a factor $[m_b^{\overline{\rm MS}}(4.2\,{\rm GeV})/m_b^{\overline{\rm MS}}(m_t)]^2 \approx 2.36$.

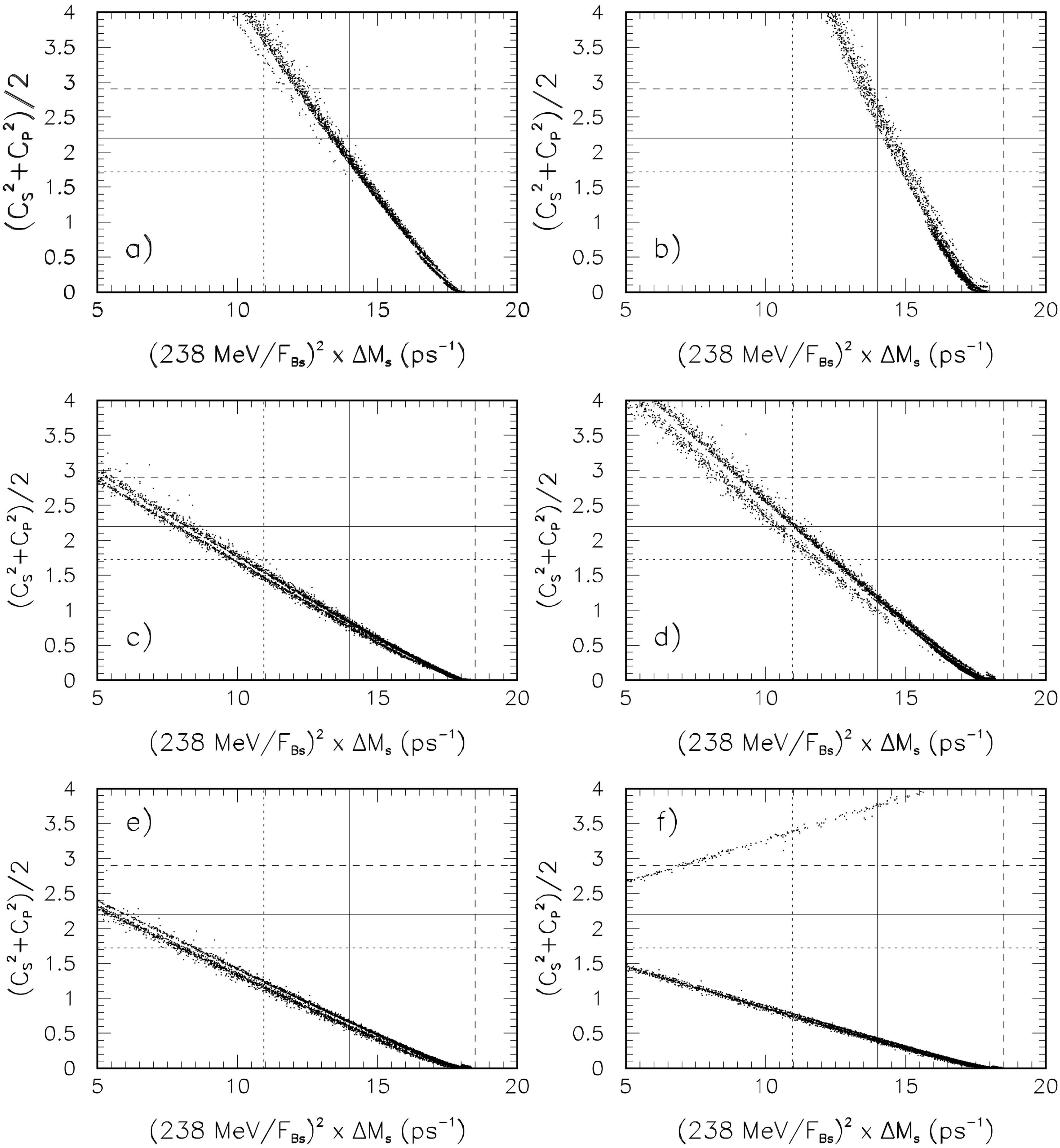

Fig. 1a–f. Scatter plots of $(1/2)\left(|C_S^{\mu(1)}|^2 + |C_P^{\mu(1)}|^2\right)$ versus ΔM_s in the MFV MSSM for sparticle masses greater than 500 GeV. Panels a–f correspond to $(M_A, \tan\beta)$ values (200,40), (200,50), (300,40), (300,50), (400,50), (500,50), respectively. Points to the left and above the solid lines are for $\hat{F}_{B_s} = 238$ MeV excluded by $\Delta M_s > 14$/ps and $\mathrm{BR}(B_s^0 \to \mu^+\mu^-) < 0.95\times10^{-6}$, respectively. The same constraints for $\hat{F}_{B_s} = 207$ MeV and 269 MeV are shown by dashed and dotted lines, respectively

and $\hat{F}_{B_s} = 238$ MeV (solid lines), $\hat{F}_{B_s} = 207$ MeV (dashed lines) and $\hat{F}_{B_s} = 269$ MeV (dotted lines). Vertical lines show the corresponding constraint imposed by the experimental lower limit $\Delta M_s > 14$/ps (excluded are the points to the left of these lines).

From Figs. 1a–f it is clear that the lowest possible values of $\hat{F}_{B_s}$ [34], which in the model independent analysis of the preceding section gave the biggest effects of the scalar operators in the $\bar{B} \to X_s\mu^+\mu^-$ and $\bar{B} \to K\mu^+\mu^-$ transitions, are in the MSSM allowed only for small values of the Wilson coefficients $|C_S^{\mu(1)}|^2$ and $|C_P^{\mu(1)}|^2$. Moreover, for $M_A > 200$ GeV and $\hat{F}_{B_s} \gtrsim 238$ MeV the lower limit $\Delta M_s > 14$/ps becomes more constraining than the bound (1). This means that in the MSSM (or any other model in which $\mathcal{O}_{S,P}$ arise from the FV couplings similar to $[X_{\mathrm{LR}}]^{qb}$ in (20)) the possible effects of the scalar operators $\mathcal{O}_{S,P}$ in the $\bar{B} \to X_s\mu^+\mu^-$ and $\bar{B} \to K\mu^+\mu^-$ decays must be smaller than the estimates given in Sect. 3. For example, using the formulae of Sect. 3 and the numbers that can be extracted from Fig. 1b, we find that for $M_A = 300$ GeV and $\tan\beta = 50$ the maximal effects in the inclusive process are bounded by

$$\Delta\mathrm{BR}(B \to X_s\mu^+\mu^-)_{\mathrm{non-res}} \lesssim 2.2 \times 10^{-7} \qquad (42)$$

obtained for $f \approx 12/14$ (for which (1) and $\Delta M_s > 14/\mathrm{ps}$ allow for the maximal value of $|\tilde{C}^\mu_S| \approx |\tilde{C}^\mu_P|$) and $r \approx 1$, that is, any effects of the scalar operators must be below 5%. The maximal effects in the exclusive decay $\bar{B} \to K\mu^+\mu^-$ are then also suppressed by the limit on the B^0_s–$\bar{B}^0_s$ mass difference:

$$\Delta\mathrm{BR}(B \to K\mu^+\mu^-)_{\mathrm{non-res}} \lesssim 1.0 \times 10^{-7} \ . \qquad (43)$$

The suppression further with decreasing value of $\tan\beta$ and increasing mass scale of the Higgs boson sector (set by M_A) up to $M_A \gtrsim 650$ GeV.

Since the effects of the FV couplings (19) in $\mathrm{BR}(B^0 \to \mu^+\mu^-)$ scale as $(1/M_A)^4$ while in ΔM_s only as $(1/M_A)^2$, for sufficiently heavy Higgs sector and sufficiently large couplings $[X_{\mathrm{LR}}]^{sb}$ it is possible to get from the formula (41) $\delta(\Delta M_s) < 2(\Delta M_s)^{\mathrm{SM}}$, that is, $|\Delta M_s|^{\mathrm{MSSM}}$ again compatible with the experimental lower limit (this possibility is seen in the upper branch of points in Fig. 1f) and, at the same time, $\mathrm{BR}(B^0 \to \mu^+\mu^-)$ below the CDF upper limit. This happens only for $M_A \gtrsim 750$ GeV. For such Higgs boson masses and values of the couplings $[X_{\mathrm{LR}}]^{sb}$ the upper bound (26) can be saturated and simultaneously $\hat{F}_{B_s}$ can assume lowest possible values obtained from lattice simulations [34]. Only then could the effects of the scalar operators $\mathcal{O}_{S,P}$ in $\bar{B} \to X_s\mu^+\mu^-$ and $\bar{B} \to K\mu^+\mu^-$ decays reach the maximal values discussed in Sect. 3 (reduced only slightly by the fact that in the MSSM $r = r' = 1$). One should stress, however, that, at least in the MFV supersymmetry, the couplings $[X_{\mathrm{LR}}]^{sb}$ of the required magnitude can be generated by the chargino–stop loops only for very large values of the stop mixing parameter A_t along with significantly split stop masses and are very unlikely from the point of view of generation of the soft SUSY breaking terms and most likely leading to the dangerous color breaking minima of the scalar fields potential.

5 Conclusions

Rare decays of B mesons are among the places where the ongoing experimental measurements can reveal the effects of new physics. The processes involving the $b \to sl^+l^-$ and $b \to dl^+l^-$ transitions are particularly interesting in this context. The most general low energy Hamiltonian describing their phenomenology involves the so-called scalar operators $\mathcal{O}^l_S = (\bar{s}_{\mathrm{L}}b_{\mathrm{R}})(\bar{l}l)$, $\mathcal{O}^l_P = (\bar{s}_{\mathrm{L}}b_{\mathrm{R}})(\bar{l}\gamma^5 l)$ (and similar ones with $s_{\mathrm{L}} \to d_{\mathrm{L}}$). Their Wilson coefficients are negligible in the SM but in models of new physics can be quite substantial compared to the coefficients of the other operators that are usually studied. This is so for example in the minimal supersymmetric extension of the SM even if the supersymmetric partners of the known particles are rather heavy, provided the ratio of the vacuum expectation values $v_u/v_d = \tan\beta$ of the two Higgs doublets is large and the mass scale of the extended Higgs sector is not too high.

In Sect. 3 of this paper, following the earlier work [13], we have used the experimental upper limits on the branching fractions $\mathrm{BR}(B^0_{s,d} \to l^+l^-)$ to place the constraints on the Wilson coefficients of the scalar operators relevant for the $b \to sl^+l^-$ and $b \to dl^+l^-$ transitions. A particularly stringent constraint obtained from the limit $\mathrm{BR}(B^0_s \to \mu^+\mu^-) < 0.95 \times 10^{-6}$ has been subsequently used to assess in a model independent way the impact the scalar operators $(\bar{s}_{\mathrm{L}}b_{\mathrm{R}})(\bar{\mu}\mu)$, $(\bar{s}_{\mathrm{L}}b_{\mathrm{R}})(\bar{\mu}\gamma^5\mu)$ may have on the rates of the inclusive $\bar{B} \to X_s\mu^+\mu^-$ and exclusive $\bar{B} \to K\mu^+\mu^-$ decays.

We have found that the increase of $\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-)$ due to the scalar operators cannot exceed (5–15)% (depending on the range of the dimuon invariant mass), that is, it is always smaller than the uncertainty of SM NNLO result which we have estimated in Sect. 2. The large effects of the scalar operators found in this decay in [29] are therefore already excluded. On the other hand, the maximal increase of the exclusive rate $\mathrm{BR}(\bar{B} \to K\mu^+\mu^-)$ can still be quite large, of order 1.7×10^{-7}, comparable with the present error of the experimental result. The latter, when compared to the SM prediction $\mathrm{BR}(\bar{B} \to K\mu^+\mu^-) = (3.5 \pm 1.2) \times 10^{-7}$ [28], leaves some room for a positive new physics contribution. However the SM prediction for this rate hinges on the theoretical problems related to the determination of the relevant non-perturbative form factors. Before this issue is settled (and the experimental errors shrink) no firm conclusion about the detectability of new physics effects in the exclusive decay $\bar{B} \to K\mu^+\mu^-$ can be drawn.

In the supersymmetric scenario with large $\tan\beta$ and not too heavy Higgs sector, in which large values of the Wilson coefficients of the scalar operators can be naturally generated, the potential effects of $\mathcal{O}^\mu_S$ and $\mathcal{O}^\mu_P$ in $b \to s\mu^+\mu^-$ are further constrained by the experimental lower limit on the B^0_s–$\bar{B}^0_s$ mass difference. This has been illustrated in Sect. 4 in the case of the minimal flavor violation scenario considered in [9,17,20]. However, the limits on the scalar operator contributions to $\mathrm{BR}(B^0_s \to \mu^+\mu^-)$, $\mathrm{BR}(\bar{B} \to X_s\mu^+\mu^-)$ and $\mathrm{BR}(\bar{B} \to K\mu^+\mu^-)$ that can be derived by inserting in the formulae of Sect. 3 numbers extracted from Fig. 1 (for different values of $\tan\beta$ and M_A) are valid also if the flavor violation originates in the squark sector, provided supersymmetric particles are heavy enough in order not to contribute appreciably to the box and vector boson penguin amplitudes. This is because the specific relation between the Wilson coefficients of $\mathcal{O}^\mu_S$ and $\mathcal{O}^\mu_P$ and the Wilson coefficients of the scalar operators contributing to the B^0_s–$\bar{B}^0_s$ mixing amplitude relies only on the existence in the low energy effective theory of the flavor violating couplings (19) and not on the specific mechanism of the flavor violation in the underlying theory.

Note added. While completing this paper we have learned about a similar independent study by Krüger et al. [57]. In particular they confirm our conclusion that the large effects of the scalar operators found in [29] in the inclusive rate are already excluded by the experimental data.

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